STAT547C: Topics in Probability for Statistics **Winter Term 1 (Fall) 2022-23**

Assignment 1

Due date: Sept. 28, 11:59 pm. *Instructor: Benjamin Bloem-Reddy*

Instructions

Academic integrity policy: I encourage you to discuss verbally with other students about the assignment. However, you should write your answers by yourself. For example, copying (either manually or electronically) part of a function or a LaTeX equation is not permitted; and if you use online resources, you must cite them. If you discuss the assignment with anyone (a classmate or anyone else), you must say so at the top of your solutions. Also, refrain from looking at answer keys from other schools, previous years, or the Math Stack Exchange. For more information, see:
`http://learningcommons.ubc.ca/guide-to-academic-integrity/`

1. *Justify your answers formally.*
2. Submit your work by 11:59 pm (Vancouver time) on the due date to Canvas (.pdf).
3. Questions marked **graded** will be graded for correctness (partial credit will be given, so make it clear what you're doing and why). The remaining questions will be marked on binary scale for "Did you make a good effort?"

1 Basics of measure theory

In all of the following, let E be a set.

1. Let $\mathcal{A} = \{A\}$, where $A \subsetneq E$. What is $\sigma\mathcal{A}$?

   [3 mark(s)]

2. Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be two σ-algebras of subsets of E. Show that $\mathcal{E}_1 \cap \mathcal{E}_2$ is also a σ-algebra.

   [3 mark(s)]

3. Give an example of two σ-algebras, $\mathcal{E}_1$ and $\mathcal{E}_2$, for which $\mathcal{E}_1 \cup \mathcal{E}_2$ is not a σ-algebra.

   [3 mark(s)]

4. Let $\mathcal{E}$ be a σ-algebra on E. Fix $D \subset E$ and let

   $$\mathcal{D} = \mathcal{E} \cap D = \{A \cap D : A \in \mathcal{E}\} .$$

   Show that $\mathcal{D}$ is a σ-algebra on D. It is called the **trace** of $\mathcal{E}$ on D, and $(D, \mathcal{D})$ is called the trace of $(E, \mathcal{E})$ on D. (Both are measurable spaces.)

   [3 mark(s)]

5. Show that a collection of sets $\mathcal{C}$ that is closed under finite intersections is also closed under countable intersections if and only if it contains the limits of all monotone *decreasing* sequences $A_1 \supset A_2 \supset \cdots$ of its sets.

[6 mark(s)]

6. Show that the Borel σ-algebra on $\mathbb{R}$ is generated by the collection of sets $\{(-\infty, a) \colon a \in \mathbb{R}\}$.

[7 mark(s)]

Question total: [25 mark(s)]

Assignment total: [25 mark(s)]