STAT547C: Topics in Probability for Statistics **Winter Term 1 (Fall) 2021-22**

Assignment 1

Due date: Sept. 22, 11:59 pm. *Instructor: Benjamin Bloem-Reddy*

Instructions

Academic integrity policy: I encourage you to discuss verbally with other students about the assignment. However, you should write your answers by yourself. For example, copying (either manually or electronically) part of a function or a LaTeX equation is not permitted; and if you use online resources, you must cite them. If you discuss the assignment with anyone (a classmate or anyone else), you must say so at the top of your solutions. Also, refrain from looking at answer keys from other schools, previous years, or the Math Stack Exchange. For more information, see:
`http://learningcommons.ubc.ca/guide-to-academic-integrity/`

1. You only have to do the required questions for full marks. While the other questions are optional, we encourage you to at least attempt them if you are motivated to learn the subject in depth.
2. *Justify your answers formally.*
3. Submit your work by 11:59 pm (Vancouver time) on the due date to Gradescope (.pdf) and Canvas (.tex).
4. Questions marked **Graded** will be graded for correctness (partial credit will be given). The remaining questions will be marked on binary scale for "Did you make a good effort?"

1 Axioms

Required: Jacod and Protter, the following exercises: 2.1, 2.4, 2.7 (**graded,** **[5 mark(s)]**), 2.8.

Optional: Jacod and Protter, 2.9-2.12.

2 Basics

Required: Jacod and Protter, 3.11-3.13

Optional: 3.4, 3.6, 3.7

3 Obtaining new distributions from old ones

Graded

One way to obtain a probability distribution is by transforming an old one. We'll see some examples in this question.

For these questions, it may be helpful to review the basic definitions of the Gamma function and the Beta function (their Wikipedia pages are sufficient for our purposes).

1. Let P denote the Poisson probability on $\mathbb{N}$ defined by

$$P(\{n\}) = p_{\lambda,n} = e^{-\lambda}\frac{\lambda^n}{n!}\ ,\ n = 0, 1, 2, \ldots$$

We can obtain a new probability by integrating λ against another function f such that $\int f(\lambda)d\lambda = 1$. For example,

$$p_{\beta,n} = \int_0^\infty p_{\lambda,n} \times \left(\beta e^{-\beta\lambda}\right)\, d\lambda\ ,\ \beta > 0,\ n = 0, 1, 2, \ldots$$

results in a valid probability on $\mathbb{N}$.

Show that the result is a valid probability. What probability does it correspond to? (We saw it as an example in class.)

[5 mark(s)]

2. Let P denote the binomial probability on $\{1, 2, \ldots, n\}$ defined for $\alpha \in [0, 1]$ by

$$P(\{k\}) = p_{\alpha,k} = \binom{n}{k}\alpha^k(1-\alpha)^{n-k}\ ,\ k = 0, 1, \ldots, n\ .$$

Let's obtain a new probability by integrating α, with $a, b \in \mathbb{N}$, as

$$p_{(a,b),k} = \int_0^1 p_{\alpha,k}\frac{(a+b-1)!}{(a-1)!(b-1)!}\alpha^{a-1}(1-\alpha)^{b-1}\, d\alpha\ .$$

Show that the result is a valid probability. What probability does it correspond to? (We saw it as an example in class.)

[5 mark(s)]

Assignment total: [15 mark(s)]